Computer Graphics and HCI Group
AG Computergrafik und HCI

FACHBEREICH INFORMATIK

Algorithmic Geometry WS 2016/2017

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Notes:
Due date: 2016-12-19 (theory)
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If you have questions or encounter any problems, feel free to visit me in room 36-218 or to send an email to karer@rhrk.uni-kl.de.

Sheet No. 2: Splines, Bezier Curves

1) Polynomial Averaging Spline (T)

For a polynomial averaging spline of 3rd order, discuss briefly what happens if:

i) $w_i \to \infty$
ii) $w_i \to 0$

2) Properties of Bezier Curves (T)

Given a control polygon C and the corresponding Bezier curve $b(C)$, let C' be the polygon created by connecting the start and the end point of C by a straight line. Prove or disprove the follwing statemens:

a) There is a control polygon C such that the area defined by C' is a convex superset of $b(C)$.

b) For every C with curve $b(C)$, C' is the convex hull of b.

c) If $H(C)$ is the convex hull of C, $b(C) \subseteq H(C)$.

3) De Casteljau Algorithm (T)

The following two-dimensional points are given:

i	0	1	2	3	4
x_i	0.0	0.2	0.6	0.8	1.0
y_i	0.0	0.4	1.2	0.8	0.2

Calculate the value of the Bezier curve at X=0.3 using the De Casteljau algorithm. Sketch the computation of the algorithm for X=0.7 and X=0.5 and also use the value you calculated to sketch the final curve.

4) Correctness of the de Casteljau Algorithm (T)

Prove the correctness of the de Casteljau algorithm. For this purpose, investigate the weight, each control point has in the computation for the point on the curve and prove that this weight is exactly the Bernstein polynomial. W.l.o.g., you can assume that the parameter space is $t \in [0, 1]$. After you proved this, you have to show that the computation of all points fulfills the definition of the Bézier curve.

5) Runtime of the de Casteljau Algorithm (T)

Calculate the runtime for the de Casteljau Algorithm for the calcultation of a single point on the curve.

6) B-Spline Curves (T)

a) Degree Elevation

For B-Splines, what are the differences between knot insertion and degree elevation?

b) Knot Vectors

Given a control polygon with five pairwise different points d_0,...,d_4, which is the minimum order of a B-Spline curve for this polygon such that it is $\mathbb{C}^2$-continous? Provide a knot vector for the Cox - De Boor Algorithm for an open and a closed curve (each at least $\mathbb{C}^2$-continous) for the control polygon. For the open case, the curve should go throuh the first and the last point of the control polygon.

c) Continuity

Can a B-Spline-Curve generated from the control polygon from part b) by the Cox - De Boor Algorithm be $\mathbb{C}^7$-continuous? Give reasons for your answer.

7) Correctness of the de Casteljau Algorithm for Surfaces (T)

Using the proof from exercise 1 of sheet 4, show that the computation is also correct for the algorithm used for points on the surface.

8) The Twist Problem (T)

What are the two variants of the twist problem for Gordon-Coons patches? What are their causes?

9) Convex Hull Property for Bezier Surfaces (T)

Prove the existence of a convex hull property for Bezier surfaces analogous to the one for Bezier curves.

10) Raycasting on B-Spline Surfaces (T)

Given a B-Spline surface and the corresponding control polygon together with a raycasting algorithm for rendering, describe an iterative algorithm to (approximatively) determine the intersection point of a given ray with that surface. Provide a feasible end condition. Give reasons for the feasibility of your end condition and the correctness of your algorithm.
Hint: If you use pseudo code, make sure that the comments allow to properly reproduce your actual idea or provide a few sentences of text explaining what is going on and mark the corresponding lines in your pseudo code.