Computer Graphics and HCI Group
AG Computergrafik und HCI

FACHBEREICH INFORMATIK

Algorithmic Geometry WS 2017/2018

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Notes:
Due date: End of semester (P), 2017-12-01 (T)
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If you have questions or encounter any problems, feel free to visit me in room 36-415 or to send an email to karer@rhrk.uni-kl.de.

Sheet No. 2: Splines, Bezier Curves

1) Polynomial Averaging Spline (T)

For a polynomial averaging spline of 3rd order, discuss briefly what happens if:

i) $w_i \to \infty$
ii) $w_i \to 0$

2) Properties of Bezier Curves (T)

Given a control polygon C and the corresponding Bezier curve $b(C)$, let C' be the polygon created by connecting the start and the end point of C by a straight line. Prove or disprove the follwing statemens:

a) There is a control polygon C such that the area defined by C' is a convex superset of $b(C)$.

b) For every C with curve $b(C)$, C' is the convex hull of b.

c) If $H(C)$ is the convex hull of C, $b(C) \subseteq H(C)$.

3) De Casteljau Algorithm (T)

The following two-dimensional points are given:

i	0	1	2	3	4
x_i	0.0	0.2	0.6	0.8	1.0
y_i	0.0	0.4	1.2	0.8	0.2

Calculate the value of the Bezier curve at X=0.3 using the De Casteljau algorithm. Sketch the computation of the algorithm for X=0.7 and X=0.5 and also use the value you calculated to sketch the final curve.

4) Correctness of the de Casteljau Algorithm (T)

Prove the correctness of the de Casteljau algorithm. For this purpose, investigate the weight, each control point has in the computation for the point on the curve and prove that this weight is exactly the Bernstein polynomial. W.l.o.g., you can assume that the parameter space is $t \in [0, 1]$. After you proved this, you have to show that the computation of all points fulfills the definition of the Bézier curve.

5) Runtime of the de Casteljau Algorithm (T)

Calculate the runtime for the de Casteljau Algorithm for the calcultation of a single point on the curve.

6) B-Spline Curves (T)

a) Degree Elevation

For B-Splines, what are the differences between knot insertion and degree elevation?

b) Knot Vectors

Given a control polygon with five pairwise different points d_0,...,d_4, which is the minimum order of a B-Spline curve for this polygon such that it is $\mathbb{C}^2$-continous? Provide a knot vector for the Cox - De Boor Algorithm for an open and a closed curve (each at least $\mathbb{C}^2$-continous) for the control polygon. For the open case, the curve should go throuh the first and the last point of the control polygon.

c) Continuity

Can a B-Spline-Curve generated from the control polygon from part b) by the Cox - De Boor Algorithm be $\mathbb{C}^7$-continuous? Give reasons for your answer.