Computer Graphics and HCI Group
AG Computergrafik und HCI

FACHBEREICH INFORMATIK

Geometric Modelling Summer 2018

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Notes:
Due: 2018-05-21
Web: http://hci.uni-kl.de/teaching/geometric-modelling-ss2018/
If you have questions or encounter any problems, feel free to send an email to i_albert12@cs.uni-kl.de.

Sheet No.1: Analytic Geometry, Projective Geometry, and Affine Geometry

1) Higher Order Vector Spaces

a) Let V be a vector space. Prove that the wedge product $\vec{a} \wedge \vec{b}$ of $\vec{a} \in \Lambda^k(V)$ and $\vec{b} \in \Lambda^l(V)$ is in $\Lambda^{k+l}(V)$.

b) Using the notation of a), prove that $\vec{a} \wedge \vec{b} = (-1)^{kl}\vec{b} \wedge \vec{a}$.

c) Prove that the exterior product is associative.

2) Objects in P^2

Your task is to project objects from a given plane F through an eye point O into a canvas plane C that is a subset of P^2. Let the plane F be denoted by $z = 0$, the canvas C by $x = 0$ and the eye $O = (1, 1, 1)^T$ (as in the exercise class).

a) **Parabola**: Consider the parabola $y = x^2$ in F. Determine its image in C by perspective projection through O. Draw a sketch of the resulting image.

b) **Circle**: What is the image in C of the circle in F with radius 2 and center $(1, 1) \in F$? Draw a sketch of the resulting image.

3) Principle of Duality

Prove: Every line in a projective space is the intersection of at least three hyperplanes.
To do so, apply the principle of duality (i.e. prove the dual statement).

4) Affine Transformations

In class, we derived the rotation around the z-axis with respect to an arbitrary point. Derive the map for the rotation with respect to an arbitrary point in 3-dimensional space but around an arbitrary axis.

5) Elliptic and Hyperbolic Spaces

In the lecture, it is stated that three lines in the hyperbolic plane uniquely define a triangle. The slides show a picture of a triangle in the Poincare disk model. What would the situation look like in the Beltrami-Klein model? Does this also hold for elliptic spaces? Provide reasons for your answers.