Geometric Modelling Summer 2018

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Notes:
Due: 2018-06-04
Web: http://hci.uni-kl.de/teaching/geometric-modelling-ss2018/
If you have questions or encounter any problems, feel free to send an email to i_albert12@cs.uni-kl.de.

Sheet No.2: Curve Theory

1) Some Proofs:

Prove the following statements:

a) A planar curve is a line iff its curvature vanishes.

b) The center point of the osculating circle is stationary exactly where the curvature is stationary.

2) Arc Length Parameterization

Consider the following helix:

$$t\colon [0, 2\pi] \to \mathbb{R}^3, \quad t \longmapsto (\alpha\cos(t), \alpha\sin(t), \beta t)^T$$

Compute the arc length parameterization and prove that the normal unit vector in every curve point crosses the axis of the helix' cylinder.

3) Coordinate Lines

Consider the following helicoid:

$$\begin{aligned} &X\colon \mathbb{R}\setminus\{0\}\times\mathbb{R} \longrightarrow \mathbb{R}^3 \\ &(u, v) \longmapsto (u\cos v, u\sin v, C\cdot v) \qquad ; C \in \mathbb{R} \end{aligned}$$

Determine the surface's coordinate lines. What special kind of curves are they? Also determine the surface's unit normal.

4) Planarity of Space Curves

Prove: A space curve is planar iff its torsion vanishes.

5) Fundamental Theorem of Curve Theory

In the lecture, we stated that a curvature and torsion would uniquely determine a curve up to Euclidian motions. Convince yourself of this fact by coming up with an example for each of the following types of curve. Then try to draw your example in differently rotated and scaled 3-dimensional Cartesian coordinate systems.

a) A curve with non-vanishing curvature and vanishing torsion.

b) A curve with non-vanishing curvature and torsion.

6) Parallel Curves

Let $X, \tilde{X}$ be two regular parameterized C^3 curves with non-vanishing curvature. Moreover, for every $t \in [a, b]$, let the line connecting $X(t)$ and $\tilde{X}(t)$ be parallel to the principal normal vectors of both curves.
Prove that the distance between $X(t)$ and $\tilde{X}(t)$ and the angle between the tangent vectors of both curves are constant in t.