Computer Graphics and HCI Group
AG Computergrafik und HCI

FACHBEREICH INFORMATIK

Geometric Modelling Summer 2018

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Notes:
Due: 2018-06-18
Web: http://hci.uni-kl.de/teaching/geometric-modelling-ss2018/
If you have questions or encounter any problems, feel free to send an email to i_albert12@cs.uni-kl.de.

Sheet No.3: Surface Theory

1) Principal Curvature

Let $X\colon \mathbb{R}^2 \to \mathbb{R}^3$ be defined by:

$$X(u,v) = (u\cos v, u\sin v, hv)^T \qquad ; (u,v) \in \mathbb{R}^2$$

where $h = const. \neq 0$.
Calculate the principal curvatures and their directions.

2) Shape Operator, Mean and Gaussian Curvature

Consider the following parameterization of a torus:

$$(\varphi,\psi) \longrightarrow ((P+\rho\cos\psi)\cos\varphi, (P+\rho\cos\psi)\sin\varphi, \rho\sin\psi) \qquad , P > \rho > 0$$

Determine the shape operator (aka. Weingarten map) L, the Gaussian curvature K and the mean curvature H.

3) Fundamental Forms

Let M be a regular surface with p_u and q_u points in the tangent space U_M of M. Then, the third fundamental form is defined as

$$\mathbf{III}(p_u, q_u) = L_U(p_u) \cdot L_U(q_u)$$

where L_U is the shape operator. In terms of the first and second fundamental form, the third fundamental form is given implicitly by $0 = \mathbf{III} - 2H\mathbf{II} + K\mathbf{I}$, where H denotes the mean curvature and K the Gaussian curvature.

Let $y\colon I \to \mathbb{R}^3$ a regular path with $y(t) = (f(t), 0, g(t))^T$ and $X\colon I \times \mathbb{R} \to \mathbb{R}^3$ the rotation surface defined by:

$$X(t, v) = (f(t)\cos v, f(t)\sin v, g(t))^T$$

where $f(t) > 0$.
Calculate the fundamental forms I, II, and III.

4) Offset Curves

Let g be the following piecewise defined closed space curve:

$$\begin{aligned}
g :& \mathbb{R} \to \mathbb{R}^3 \\
X =& \quad (sin(t-1), cos(t-1), -0.5)^T - (1,0,0)^T, \text{ for} t \in [0,1] \\
X =& \quad (sin(t-2), cos(t-2), \tfrac{t-2}{4})^T, \text{ for} t \in [2,3] \\
X =& \quad (-.5, -2, .75)^T - (t-4)(-0.3, 0.2, -0.25)^T, \text{ for} t \in [4,5]
\end{aligned}$$

, where the intervals $[1, 2]$, $[3, 4]$, and '$[5, 0]$' connect the curve segments C^1-continuously by interpolating quadratic Bezier segments.

Pick at least two of these intervals and compute the offset curve obtained from applying an offset of 1 along the following directions:

- in interval $[0, 1]$: $(0, 0, 1)^T$
- in interval $[2, 3]$: 1 along the principal normal direction
- in interval $[4, 5]$: $(0, 1, 0)$
- for the rest, interpolate the offset directions in the respective start and end points beginning with the direction in the start point and rotating the vector with constant velocity (i.e. adjust the rotation angle depending on the the arc length)

Challenge: Also do this for the other segments to obtain the complete offset curve. Do *not* use computers or other technical aids. Document every step of your solution and how exactly you apply it.