Geometric Modelling Summer 2018

Prof. Dr. Hans Hagen

http://hci.uni-kl.de/teaching/geometric-modelling-ss2018

Organization

People

- Lecture: Prof Dr. Hans Hagen
  - 36-217
  - hagen@informatik.uni-kl.de
- Lecture: M.Sc. Benjamin Karer
  - 36-415
  - karer@rhrk.uni-kl.de
- Exercises: B.Sc. Iannis Albert
  - i_albert12@cs.uni-kl.de

Lecture and Exercise

- mode: 2 + 2 (5 ECTS)
- Lecture:
  - Monday, 11:45-13:45, 36-265
- Exercises:
  - to be announced

Exercises and Exam Admission

- 3 Components:
  - "homework" (**mandatory!**)
  - biweekly tutorials (not mandatory)
  - biweekly exercise lessons (not mandatory)
  - seminar talk at the end of the course (**mandatory!**)
- exam admission:
  1. exercise sheets have to be submitted until the due date
  2. short talk and paper discussion have to be done before the exam
- attendence to exercise lessons and tutorials is not mandatory but highly recommended

Exercise Sheets

- **each** task on **every** sheet has to be attempted
- no "points"
- instead, the attempt has to be reasonably documented, i.e.
  - the reasoning behind your answers has to become clear
  - the several steps that lead to your solution have to be presented clearly and well-structured
- in turn, we also accept false solutions as long as the thought that led to them becomes clear

Seminar Talk and Paper

- each student will receive one paper
- papers are handed out in the lecture or on request
- short talk:
  - 10-15 minutes talk, 5 minutes discussion
  - high-level explanation of problem, method, results
  - focus on **your personal conclusion**
  - dates to be announced at the end of the course
- seminar paper:
  - about 1 to 2 pages (including images)
  - high-level explanation of problem and solution approach
  - discussion of the paper and your own opinion about it
  - to be handed in at least 1 week before the talk

Exercise Setup

- 4 Components:
  1. discussion of the homework
  2. questions & answers on lecture content
  3. further (introductory) exercises and discussions
  4. further explanation on selected topics from the lecture
- aims:
  - provide deeper understanding and missing links
  - prepare you for the next homework and the exam
  - train your skills in scientific discussion

Literature

- M. P. do Carmo: *Differential Geometry of Curves and Surfaces*; Prentice Hall, 1976
  (there's a free version of the book available online)
- J. Hoschek, D. Lasser: *Grundlagen der geometrischen Datenverarbeitung (2. Aufl.)*; Teubner, 1992 (German textbook)
- W. Kühnel: *Differential Geometry: Curves - Surfaces - Manifolds, Second Edition*; American Mathematical Society, 2005
- K. Jänich: *Vector Analysis*; Springer, 2001

Lecture Outline:

1. Motivation
2. Foundations from Analytic Geometry
3. Foundations from Projective Geometry
4. Affine Spaces, Elliptic and Hyperbolic Geometry
5. Foundations from Vector Analysis
6. Differential Geometry I – Curve Theory
7. Differential Geometry II – Surface Theory
8. Variational Design
9. Offset Curves
10. Offset Surfaces
11. Interpolating Triangle Patches

Motivation

How can Geometry be described?

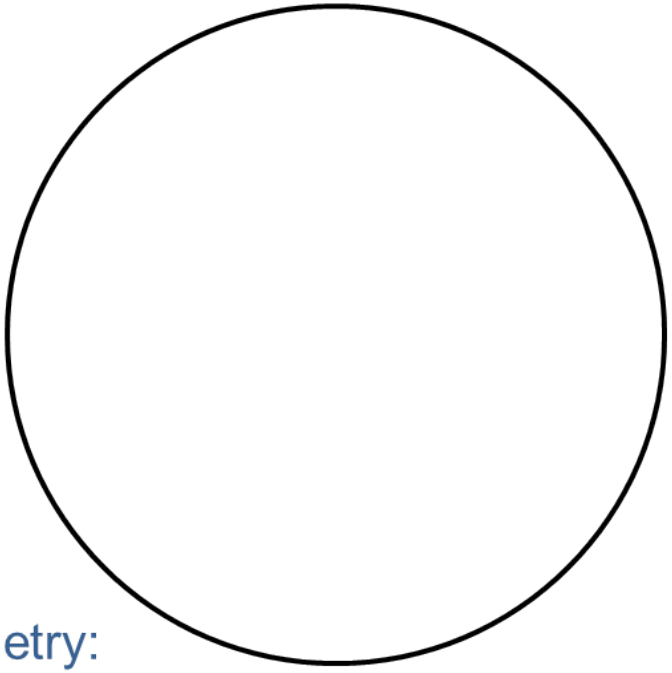
Definitions of the Circle:

Algebraic Geometry:

$$x^2 + y^2 = r^2$$

Analytic Geometry:

$$\begin{bmatrix} r\sin(\alpha) \\ r\cos(\alpha) \end{bmatrix}$$



Differential Geometry:

Curvature: $\kappa = \frac{1}{r}$

Torsion: $\tau = 0$

Foundations of Geometry:

A circle is the set of all points that have equal distance to a distinguished point.

How can Geometry be described?

Differential Geometry:

In Differential Geometry, the two scalar invariants *curvature* and *torsion* allow for a complete description of curves which is independent from the coordinate system.

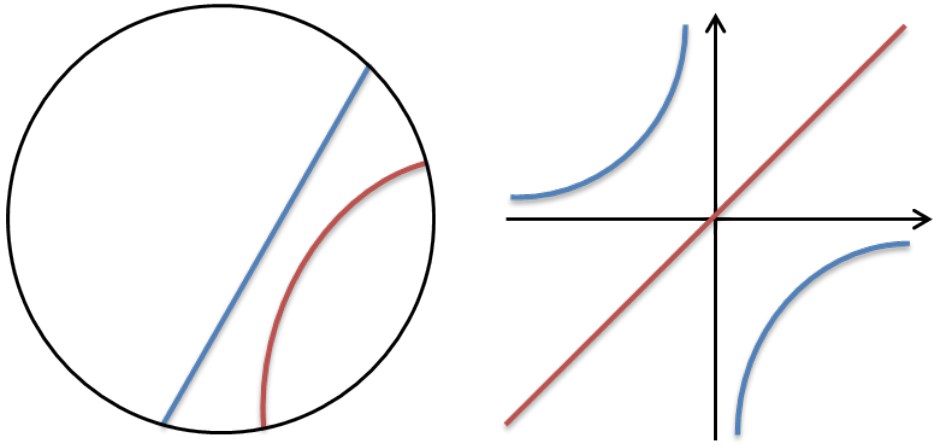


Figure: Left: Hyperbolic space in the Beltrami-Klein model. Hyperbolic straight line (blue) and line of curvature 0 (red). Right: Representations of these lines in Euclidean Space.

How can Geometry be described?

Variational Design:

The generation of "technically smooth" surfaces from a point cloud is a key problem in Computer Aided Geometric Design. Variational Design generates smooth curves and surfaces that fulfill certain constraints and minimize certain functionals which can be interpreted in the sense of physics and/or geometry.

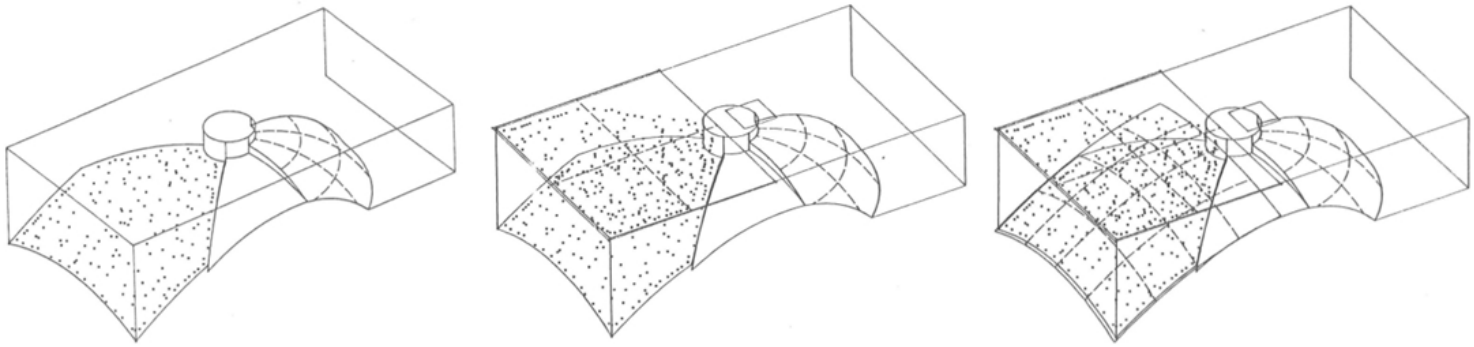


Figure: Digitalization, Parameterization, Variational Surface Design (left to right) for the reflection surface of a car's headlights.

How can Geometry be described?

Offset Surfaces:

Offset surfaces occur in various problems in Geometric Modelling, e.g. when modelling surfaces with realistic material thickness.

Interpolating Triangle Patches:

Surfaces and point-based meshes are usually modelled using quadrilateral patches. However, in many applications situations occur in which it is better to work with triangles.

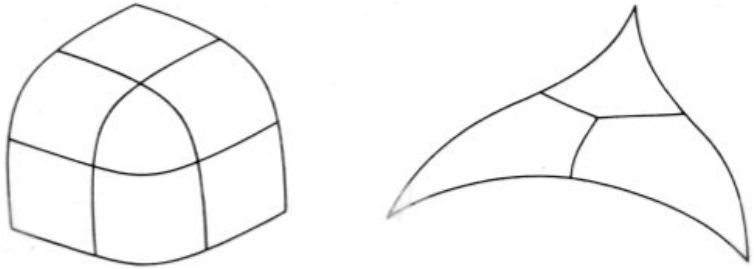


Figure: Examples for the use of interpolating triangle patches.